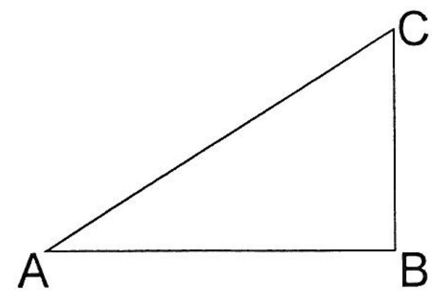
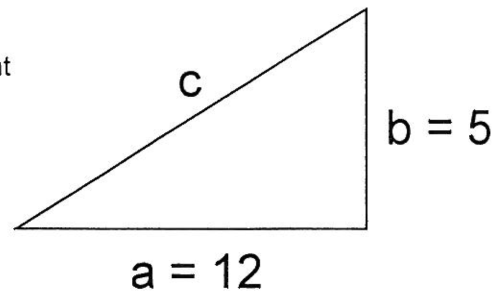
Right Triangles & Trigonometry



Suppose you wanted to get from point A to point C in the following diagram. You could go directly from A to C, or you could go to the right from A to B and then go straight up from B to C. Thus the direction we go is important. We will define the distance from C to A as our displacement. You'll see shortly that the displacement is a vector. In many cases, we will be interested in the x and y component of a vector. In this case, the x component is AB. The y component is BC.

Pythagorean Theorem



If two of the three sides of a right triangle are known, we can find the 3rd side using the Pythagorean Theorem.

Recall that $a^2 + b^2 = c^2$

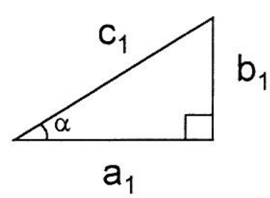
$12^2 + 5^2 = c^2$

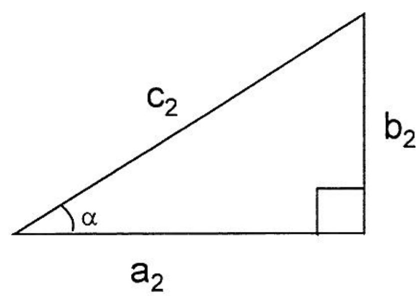
$144 + 25 = c^2$

$169 = c^2$

$13 = c$

Right Triangle Trigonometry





Both of the triangles shown are right triangles. Angle α is the same for both.

Side c is the hypotenuse. Side a is adjacent to angle α. Side b is opposite angle α. If we divided side b by side a for both triangles we would get the same number. The only way we could get a different ratio would be if the angle changed. For example, if the angle increased, side b would have to increase, while side a remained the same. That would cause the ratio to increase. We call the ratio of side b to side a the tangent of the angle. The same logic is true for the ratios of any two sides. We will use three ratios:

Sine → $\sin\alpha = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{b}{c}$

Cosine → $\cos\alpha = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{a}{c}$

Tangent → $\tan\alpha = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{b}{a}$

An easy way to remember these is the acronym **SOHCAHTOA** (pronounced so ca toe a)

SOH → **S**ine is **O**pposite / **H**ypotneuse

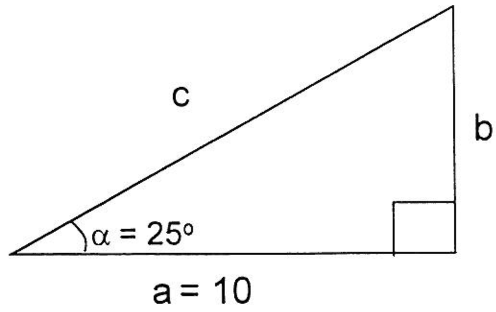
CAH → **C**osine is **A**djacent / **H**ypotenuse

TOA → **T**angent is **O**pposite / **A**djacent

Make sure your calculator is in degree mode, not radian mode.

Examples

Find sides b and c.



$$\tan \alpha = \frac{Opp}{Adj} = \frac{b}{a}$$

$$\tan 25 = \frac{b}{10}$$

$$b = 10 \tan 25 = 4.66$$

Now that we know side b, we can use trig or the Pythagorean theorem to find side c.

$$\cos \alpha = \frac{Adjacent}{Hypotenuse} = \frac{a}{c}$$

$$\cos 25 = \frac{10}{c}$$

$$c = \frac{10}{\cos 25} = 11.03$$

Verify this answer by finding c using the Pythagorean theorem.

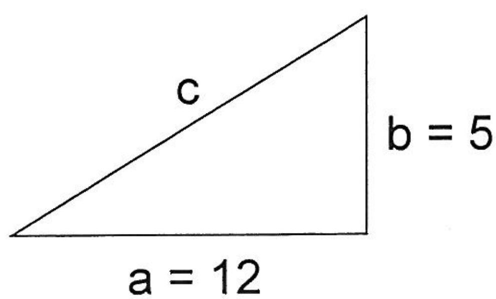
Finding angles

Since each angle has a unique sine, cosine and tangent value, we can use these values to find the angle. We call these functions the inverse tangent, inverse sine, or inverse cosine.

$$\alpha = \tan^{-1} \frac{Opposite}{Adjacent} = \tan^{-1} \frac{b}{a}$$

We'd read this equation as

"α is the angle whose tangent is b/a"



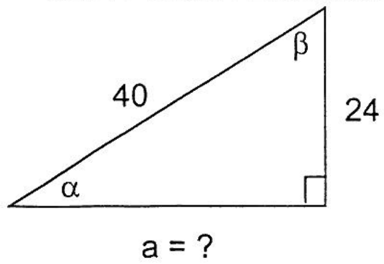
For our example,

$$\alpha = \tan^{-1} \frac{Opposite}{Adjacent} = \tan^{-1} \frac{5}{12}$$

The inverse functions are normally above (shift or 2nd function) the standard trig function button on your calculator.

$$\alpha = 22.62°$$

Name: ____



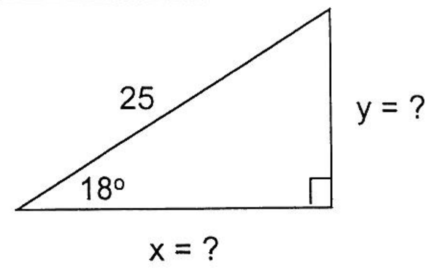
1. What is the length of side a?

3. What is the sine of angle α ?

4. What is the cosine of angle α ?

5. Angle α = ____________degrees.

3. Angle β = ____________degrees.
(use trig and check to see if all the angles add to 180°)



1. What is the length of side x?

2. What is the length of side y?

3. Use the Pythagorean theorem to check your answers.

If the 25 was a vector quantity (see chapter 1), the values you found for x and y would be called the x and y components.